Topics for talks
Spring 2018
Convex and Algebraic Geometry Seminar

Topics are organized into four blocks (Newton–Okounkov convex bodies, Schubert calculus, Geometry of Schubert varieties, Lattice polytopes and their applications). Every topic is intended for a single talk. The order of topics inside every block is fixed, that is, later talks in the same block rely on earlier talks. However, blocks can be interchanged.

Newton–Okounkov convex bodies.

(1) Line bundles, the degree of a projective variety, Hilbert polynomial [GH, 1.1],[Ha, 1.7]
(2) Kouchnirenko's theorem and its proof via Hilbert's theorem [Kh]
(3) Graded semigroups of lattice points in $\mathbb{R}^n$ and approximation theorem [KaKh, Part 1]
(4) Newton–Okounkov convex bodies and generalizations of Kouchnirenko's theorem to non-toric varieties [KaKh, LM, KST]

Schubert calculus

(1) Schubert's problem on 4 lines and its solutions via hyperboloid and Grassmannian [Reid, 7.5], [Ro, KL], Schubert calculus and Hilbert's 15th problem [Kl76, Kl85]
(2) Grassmannians [KL], [Ma, 3.1-3.2] and Littlewood–Richardson rule [Ma, 1.5], [Fu, Va]
(3) Littlewood–Richardson rule and Knutson–Tao puzzles [CV, Kn12]
(4) Flag varieties and Schubert polynomials [B, Ma, Fu]
(5) Pipe dreams, mitosis and Fomin–Kirillov theorem [FK, Mi, Ma]

Geometry of Schubert varieties

(1) Defining Grassmannians by Plücker relations. [Ma, 3.1–3.2]
(2) Coordinate rings of Schubert varieties and standard monomials. [Ma, 3.3]
(3) Resolutions of singularities for Schubert varieties. Small resolutions. [Ma, Z].
(4) Singular loci of Schubert varieties. [Ma, 3.4]

Lattice polytopes and their applications

(1) Multidimensional Pick's formula and the Khovanskii–Pukhlikov theorem [BR, 10.1-10.4], [CLS, §13.4-13.5]
(2) Gelfand–Zetlin bases and polytopes for the representations of $GL_n(\mathbb{C})$ (with detailed examples in the case $n = 3$) [FH, Mo]
(3) Comparison of Gelfand–Zetlin polytopes and Feigin–Fourier–Littelmann–Vinberg polytopes [ABS, Ki17, Fo]
(4) Symplectic Gelfand–Zetlin polytope and Okounkov's theorem [O]

Below are references available in English (with comments if a Russian version is available). If you read Russian ask the organizers for additional references available only in Russian.

To learn more about recent developments in algebraic geometry and representation theory that are connected directly with the topics listed above you are encouraged to read [FaFL, FFL, HY, Ka, Ki16].

References

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