Mathematical Computations

Spring 2018, Faculty of Mathematics NRU HSE

Exercises

1. Numbers

Problem 1.1. Find the 100-th digit after the decimal point in the decimal fraction of π.

Problem 1.2. Calculate $e^{\pi\sqrt{163}}$ up to 12 digits after the decimal point. Subtract 744 and extract the cubic root. (The result should be very close to an integer number[1]. This can also be justified using number theory, for a popularizing explanation see [163]. A positive integer numbers d is called a *Heegner number* if the ring of integers of the quadratic extension $\mathbb{Q}(\sqrt{-d})$ is a Unique Factorization Domain. It turns out that 163 is the largest Heegner number.)

Problem 1.3. Find the first 100 terms of the continued fraction decomposition for e. Guess an explicit formula for the n-th term. (There is an elegant proof using integration by parts [Cohn].)

2. Plane geometry

Problem 2.1. Program a manipulator that draws a regular n-gon for a given n from 3 to 100.

3. Geometry in 3-space

Problem 3.1. Draw a polytope of the same shape as the pyramid of Cheops.

Problem 3.2. Draw two families of lines on a section of the Shukhov tower. (Geometrically, every section is part of a hyperboloid. For a popularizing explanation on how to construct two families of lines see [GeomArch].)

4. Differential equations

Problem 4.1. Make an animation of a vibrating string.

References

[Cohn] Henry Cohn, *A Short Proof of the Simple Continued Fraction Expansion of e*, The American Mathematical Monthly, **113** (2006), no. 1, 57–62

[163] *163 and the Ramanujan constant*, video by Numberphile, `https://www.youtube.com/watch?v=DRxAVA6gYMM`

[GeomArch] *Geometry for architecture*, video by ShinyMath, `https://www.youtube.com/watch?v=_BpIZqKlHdw` [In Russian]

[1] In 1975, Martin Gardner made an April Fool's joke claiming that this number is integer.