Mathematical Computations

Spring 2018, Faculty of Mathematics NRU HSE

Exercises for 09.04.2018

1. Numbers

Problem 1.1. (a) Find the binary and the hexadecimal decompositions of 1000000.
(b) Find out which of the fractions $\frac{1}{n}$ for $n = 1, \ldots, 40$ lie in the Cantor ternary set.

Problem 1.2. (a) Find k such that the k-th convergent of the continued fraction for $\sqrt{2}$ is equal to $\frac{297}{210}$. (Recall that the standard A4 size paper measures 210 mm wide and 297 mm long. For the mathematics behind this choice, see, e.g., [Kvantik].)
(b) Find the first 10 convergents of π.

Problem 1.3. Find the 300-th digit of the number 300!. Which digit occurs most frequently among all digits of this number? Which digit is the least frequent?

2. Combinatorics

Problem 2.1. List all partitions of a given number n into parts of
(a) odd lengths;
(b) pairwise different lengths.
(A *partition* of n is the decomposition $n = k_1 + \ldots + k_s$, where $k_1 \geq \ldots \geq k_s$ are positive integers). Check that both lists have the same number of elements.

3. Equations

Problem 3.1. Show that a regular 17-gon can be constructed by a compass and straightedge. Equivalently, solve by radicals the equation $x^{17} - 1 = 0$.

4. Plane geometry

Problem 4.1. Draw a connected but not path connected subset on the plane. (One example is the union of the graph $\{(x, \sin \frac{1}{x}) \mid x \in (0, 1)\}$ with the segment $[(0, -1), (0, 1)]$ on the y-axis.)

5. Polynomials

Problem 5.1. Find all irreducible polynomials of degree 5 over the field of 3 elements.

6. Matrices

Problem 6.1. For the matrix:

$$A = \begin{pmatrix} 5 & 4 & 2 & 1 \\ 0 & 1 & -1 & -1 \\ -1 & -1 & 3 & 0 \\ 1 & 1 & -1 & 2 \end{pmatrix},$$

find

(a) A^{20};
(b) characteristic and minimal polynomials;
(c) Jordan normal form;
(d) eigenvalues and eigenvectors.

References

[Kvantik] Evgeny Smirnov, *Arithmetics of a sheet of paper* [In Russian], Kvantik, 2017, no. 1, 8–10