

Mathematical Computations  
SPRING 2018, FACULTY OF MATHEMATICS NRU HSE

**Exercises for 16.04.2018**

1. NUMBERS

*Problem 1.1.* (a) Given a list of numbers, find the sum and product of all elements in this list.  
(b) Find the sum of the first 100 prime numbers.

*Problem 1.2.* In [RSA], Rivest, Shamir and Adleman offered the following challenge. A plain text message is encoded by replacing  $A$  by 01,  $B$  by 02 etc. The space between words is encoded by 00. Let  $x$  denote the resulting number, and  $n = pq$  the product of two primes

$$p = 32769132993266709549961988190834461413177642967992942539798288533$$

$$q = 3490529510847650949147849619903898133417764638493387843990820577.$$

It is known that  $x^{9007} \pmod{n}$  is equal to:

9686 9613 7546 2206 1477 1409 2225 4355

8829 0575 9991 1245 7431 9874 6951 2093

0816 2982 2514 5708 3569 3147 6622 8839

8962 8013 3919 9055 1829 9451 5781 5154

Find  $x$  and the plain text message. (In the original challenge, the decomposition of  $n$  into a product of primes was not given. Finding  $p$  and  $q$  was the main part of the challenge. Even now, Mathematica is not able to decompose  $n$  into primes in reasonable amount of time.)

2. COMBINATORICS

*Problem 2.1.* Construct the perfect binary tree of depth 4 using TreeForm.

3. GEOMETRY IN 3-SPACE

*Problem 3.1.* Draw a net for folding each of the five Platonic solids.

4. FUNCTIONS

*Problem 4.1.* Define the function  $f(n)$  whose value at  $n \in \mathbb{N}$  is equal to the  $n$ -th Fibonacci number.

*Problem 4.2.* (a) Draw the graphs of the first ten Chebyshev polynomials.

(b) Express  $\cos nx$  as a polynomial in  $\cos x$ .

*Problem 4.3.* Draw the graph of the function  $\sin(x + y) + \cos(xy)$ .

*Problem 4.4.* Draw level sets of the function  $f \circ A^n$ , where

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \quad f(x, y) = \sin x + \sin y.$$

*Problem 4.5.* Draw the image of the coordinate grid under Joukowski map  $z \mapsto z + \frac{1}{z}$ .

#### REFERENCES

[RSA] M.GARDNER, *A new kind of cipher that would take millions of years to break*, Scientific American, August 1977, 120–124