

Mathematical Computations
SPRING 2018, FACULTY OF MATHEMATICS NRU HSE
Exercises for 23.04.2018

1. COMBINATORICS

Problem 1.1. Program a manipulator that draws complete graphs with n vertices for all n from 1 to 10.

2. GEOMETRY IN 3-SPACE

Problem 2.1. Draw the Möbius band.

Problem 2.2. Draw the Boy surface. (The Boy surface is an immersion of \mathbb{RP}^2 into \mathbb{R}^3 . It can be constructed by gluing opposite points of the boundary of a hemisphere. This process is visualized in [1]. In particular, a planet shaped as the Boy surface would have the North pole but no South pole [2].)

3. GROUPS

Problem 3.1. Find the number of Abelian groups of order 1000.

Problem 3.2. Write down the multiplication table for the quaternion group $Q_8 = \{\pm 1, \pm i, \pm j, \pm k\}$.

Problem 3.3. Find the order of the Monster group (the largest sporadic simple group) and decompose it into prime factors.

4. POLYNOMIALS

Problem 4.1. Generate a list of all Laurent polynomials in two variables with coefficients 1 whose Newton polygons lie inside the square with vertices $(\pm 1, \pm 1)$, $(\pm 1, \mp 1)$.

Problem 4.2. Find the first cyclotomic polynomial that has a nonzero coefficient $a \neq \pm 1$. Find the value of a .

5. FUNCTIONS

Problem 5.1. Define the Fourier transform \hat{f} of the function $f : \mathbb{R} \rightarrow \mathbb{C}$ by the formula

$$\hat{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt.$$

Find the Fourier transform of the functions:

(a) e^{-t^2} ; (b) $\operatorname{sech}(t)$; (c) t .

Problem 5.2. Let V be an n -dimensional vector space. Recall that there is a direct sum decomposition $V^{\otimes 2} = \operatorname{Sym}^2 V + \Lambda^2 V$. Define functions that assign to a tensor in $V^{\otimes 2}$ the corresponding symmetric and skew-symmetric summands.

REFERENCES

- [1] [Boy1] JOS LEYS, *The Boy surface*, animation https://www.youtube.com/watch?v=uiq-EcQz_uU
- [2] [Boy2] J.-P PETIT, *Topo the world*, cartoon story