

**Test September 9: Variant I**

FACULTY OF MATHEMATICS, HSE UNIVERSITY

Last name, first name:

<i>Problem</i>	1	2	3	4	5	6	Total
<i>Score</i>							

Test duration is 80 minutes. To get a perfect score it is enough to solve any five problems. Please write neatly. You may only use pen and paper. It is forbidden to leave the room during the test.

**Problem 1.** Decompose the polynomial

$$x^4 - 3x^2 + 9$$

into product of irreducible polynomials with rational coefficients.

**Problem 2.** A counterfeiter produced a hundred of 9 dollar bills and a hundred of 13 dollar bills. How many ways are there to pay 530 dollars using his bills?

**Problem 3.** Rationalize the denominator

$$\frac{1}{(2 + \sqrt[3]{3} + \sqrt[3]{9})}$$

**Problem 4.** Find all complex solutions of the equation:

$$z^4 + 16 = 0.$$

**Problem 5.** Find real numbers  $a$ ,  $b$ ,  $c$  and  $d$ , such that the following identity holds:

$$\frac{x^3 + 3}{(x - 1)(x - 2)(x - 3)} = a + \frac{b}{x - 1} + \frac{c}{x - 2} + \frac{d}{x - 3}.$$

**Problem 6.** Find all irreducible polynomials of degree four over the field of two elements.